LIFT HEIGHT OF VAPOR BUBBLES IN AN UNDERHEATED LIQUID IN THE CASE OF BOILING ON VERTICAL WALLS

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The average lift height of vapor bubbles in underheated alcohol boiling on vertical aluminum walls is considered. Using the Ozeen [5] approximation the lift velocity of the gas in the liquid is calculated from Reynolds numbers up to ~ 20, and the rate of collapse of a vapor bubble is estimated.

It has now been fully established that the generation, growth, and detachment of vapor bubbles on a heated wall bears a statistical character [1, 2], while the parameters defining the statistical distribution depend on the wall material, the treatment applied to the latter, the type of liquid, its mode of preparation, the heat flows involved, the thermodynamic parameters, the degree of underheating of the liquid, the wall temperature, and so on. Surface boiling may be divided into two modes, respectively characterized by moderate and substantial underheating. For moderate underheating the vapor bubbles formed on the walls detach themselves and pass into the liquid, where as a result of condensation they gradually collapse. For a high degree of underheating the layer of heated liquid is so thin that the developing bubbles entirely fail to escape, and vanish while still on the wall.

In this paper we shall consider the first proven. The greatest diameter of escaping bubbles in the boiling of an underheated liquid is ~1 mm [2]. The lift velocity may reach ~30 cm/sec and the Reynolds number $Re = Ur_0/\nu \leqslant 30$, i.e., the Stokes solution is inapplicable. A solution to the problem of a single gas bubble moving in a liquid was obtained in [3] on the basis of the method of coalescing asymptotic expansions [4]. However, the number of approximations employed appears to have been insufficient, and this led to a physically inexplicable result: for $Re \geqslant 2.5$ the resistance to the motion started increasing (Fig. 1).

In order to estimate the rate of flotation of a single bubble in an infinite liquid we shall confine attention to the Ozeen approximation [5]: a flow of viscous liquid is incident upon a spherical bubble of radius r_0 at a velocity U. We shall pay no attention to the change in the size of the bubble due to condensation; we shall take the corresponding correction factor for the resistance from [6]. The motion inside and outside the bubble is respectively described by the equations

$$\operatorname{grad} p_1 = \mu_1 \Delta \vec{v}_1, \ \operatorname{div} \vec{v}_1 = 0, \tag{1}$$

$$U \frac{\partial v}{\partial x} = -\frac{1}{\rho} \operatorname{grad} p + \nu \Delta \vec{v}, \ \operatorname{div} \vec{v} = 0. \tag{2}$$

We employ the conventional notation, the index 1 signifying quantities relating to the vapor inside a bubble or the liquid inside a drop. In spherical coordinates the boundary conditions will be

$$\begin{gathered} v_r = U\cos\theta, \ v_\theta = -U\sin\theta \ \text{for} \ r \to \infty, \\ v_r = v_{1r} = 0, \ v_\theta = v_{1\theta}, \ p_{rr} = p_{1rr}, \ p_{r\theta} = p_{1r\theta} \ \text{for} \ r = r_0, \end{gathered} \tag{3}$$

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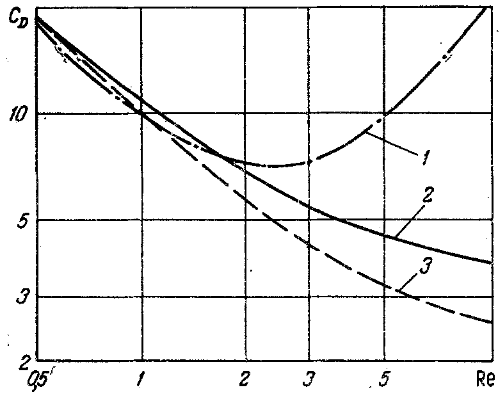

Fig. 1

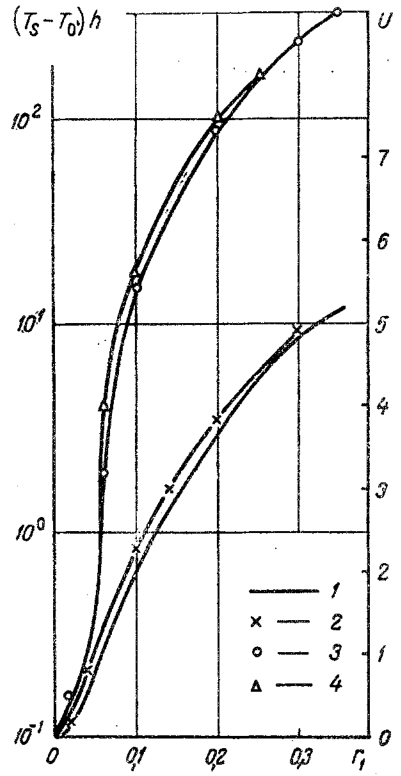

Fig. 2

Fig. 1. Resistance of a spherical gas bubble for motion in a liquid: 1) Solution of [3]; 2) present solution; 3) experiment.

Fig. 2. Lift height and velocity of a vapor bubble in alcohol and water: 1) Velocity in water (cm/sec); 2) in alcohol (cm/sec); 3) lift height in water (cm·deg); 4) in alcohol (cm·deg).

the solution being bounded in the center of the drop or bubble; p_{rr} and $p_{r\theta}$ denote the corresponding components of the stress tensor. We seek the solution of Eq. (1) in the form

$$v_{1r}(r,\ \theta) = f(r)\cos\theta,\ v_{1\theta}(r,\ \theta) = -g(r)\sin\theta,\ p_1(r,\ \theta) = \mu_1 h(r)\cos\theta$$

and we obtain

$$f(r) = \frac{a_3}{r^3} + \frac{a_2}{r} + a_0 + a_1 r^2,$$

$$g(r) = -\frac{a_3}{2r^3} + \frac{a_2}{2r} + a_0 + 2a_1 r^2,$$

$$h(r) = \frac{a_2}{r^2} + 10a_1 r,$$

with due allowance for the boundary conditions

$$f(r) = -a_1(r_0^2 - r^2),\ g(r) = a_1(2r^2 - r_0^2),\ h(r) = 10a_1 r.$$

It was shown in [5] that the solution of Eq. (2) was

$$v_r = \frac{\partial\varphi}{\partial r} + \frac{1}{2k}\frac{\partial\chi}{\partial r} - \chi\cos\theta,\ v_\theta = \frac{1}{r}\frac{\partial\varphi}{\partial\theta} + \frac{1}{2kr}\frac{\partial\chi}{\partial\theta} + \chi\sin\theta, \tag{4}$$

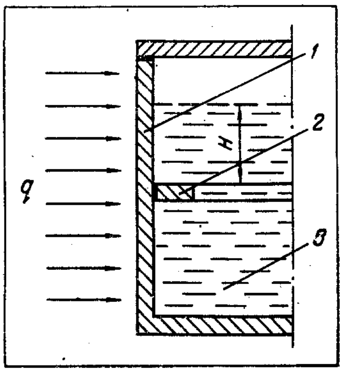

Fig. 3

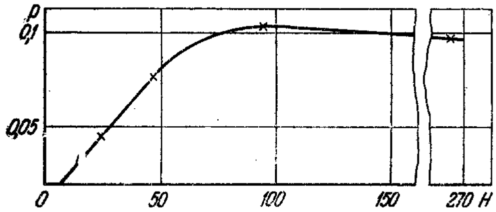

Fig. 4

Fig. 3. Arrangement of the experiments: 1) tank containing the liquid; 2) annular screen; 3) liquid.

Fig. 4. Pressure increment p (kg/cm^2) in the tank as a function of the height H(mm) at which the annular barrier is situated.

$$v_\lambda = 0,\ p = p_0 - \rho U \frac{\partial \varphi}{\partial x}, \tag{4a}$$

where

$$\varphi = \frac{A_0}{r} + A_1 \frac{\partial}{\partial x}\left(\frac{1}{r}\right) + A_2 \frac{\partial^2}{\partial x^2}\left(\frac{1}{r}\right) + \ldots, \tag{5}$$

$$\chi = -U + e^{kx}\left\{C_0 \frac{\exp(-kr)}{r} + C_1 \frac{\partial}{\partial x}\left[\frac{\exp(-kr)}{r}\right] + C_2 \frac{\partial^2}{\partial x^2}\left[\frac{\exp(-kr)}{r}\right] + \ldots\right\}, \tag{6}$$

A_i, C_i are constant $k = U/2\nu$.

In the Ozeen approximation the solution allows for the second terms of the expansions in terms of the Reynolds number. If, therefore, we carry out the expansions in terms of Re in Eq. (6) and retain terms with coefficients of the order of Re in (5) and (6), we shall approximately obtain

$$\varphi \approx \frac{A_0}{r} - \frac{A_1 \cos\theta}{r^2} + \frac{A_2(3\cos^2\theta - 1)}{r^3}, \tag{5a}$$

$$\chi \approx -U + C_0\left[\frac{1}{r} - k(1-\cos\theta) + \frac{k^2 r}{2}(1-\cos\theta)^2\right] - C_1\left(\frac{\cos\theta}{r^2} + \frac{k\cos^2\theta}{r}\right) + C_2 \frac{3\cos^2\theta - 1}{r^3}. \tag{6a}$$

Substituting (5a) and (6a) into (4) and (4a), and the resultant values of the velocities and the pressure (together with the velocities and pressure inside the bubble) into Eq. (3), and subsequently equating the coefficients or $\cos^1\theta$ and $(3\cos^2\theta - 1)$ in the resultant expressions, we obtain a system of linear equations for determining A_i, C_i, a_1:

$$a_1 = \frac{U}{2r_0^2} \cdot \frac{\mu}{\mu + \mu_1},\ C_0 = \frac{3Ur_0 - 2a_1 r_0^3}{2 - \frac{3}{2}kr_0},\ C_1 = kr_0^2 C_0,$$
$$A_0 = \frac{1}{2}\left(kr_0^2 - \frac{1}{k}\right)C_0,\ A_1 = -\frac{r_0^3}{2}\left(U + \frac{k}{2}C_0\right). \tag{7}$$

A_2 and C_2 enter into a single equation and to the degree of approximation assumed are not separated from one another; as in the Ozeen solution, we take no further note of the corresponding terms in (5) and (6). Substituting a_1 and C_0 we obtain

$$C_0 = C_0^* \frac{2\mu + 3\mu_1}{3\mu + 3\mu_1}.$$

Here C_0^* is the value of C_0 for a solid sphere. In extreme cases, in which we consider the motion of a liquid drop in an incompressible gas $\mu_1 >> \mu$, so that $\alpha_1 \approx 0$, $C_0 \approx C_0^*$ and the flow outside the drop coincides with the flow around a solid sphere for the same Re number; when we consider the motion of a gas bubble in a liquid $\mu >> \mu_1$, so that $\alpha_1 = U/2r_0^2$, $C_0 = {}^2/_3\, C_0^*$ and the difference relative to the motion of a solid sphere will be at its greatest. The velocities associated with the motion of a bubble, expressed in terms of the coefficients A_i, C_i, coincides with the analogous expressions for the motion of solid spheres [5]; on the surface of the bubble $\mu \sin\theta/(\mu + \mu_1)$ and may equal half the velocity of the center of the bubble.

The resistance of the bubble is given by:

$$F = \int_0^\pi (p_{rr}\cos\theta - p_{r\theta}\sin\theta)_{r=r_0} 2\pi r_0^2 \sin\theta d\theta, \qquad p_{rr} = -p + 2\mu\left(\frac{\partial v_r}{\partial r}\right). \tag{8}$$

From the equation of continuity

$$\left(\frac{\partial v_r}{\partial r}\right)_{r=r_0} = -\frac{1}{r_0}\left(\frac{\partial v_\theta}{\partial\theta} + v_\theta \operatorname{ctg}\theta\right)_{r=r_0}. \tag{9}$$

Since $v_r = 0$ over the whole surface of the bubble $\partial v_r/\partial\theta = 0$, and from the expression for the projection of the vortex $\Omega_\lambda = v_\theta/r + \partial v_\theta/\partial r - 1/r(\partial v_r/\partial\theta)(\Omega_r = \Omega_\theta = 0)$, on the drop surface

$$\left(\frac{\partial v_\theta}{\partial r}\right)_{r=r_0} = \left(\Omega_\lambda - \frac{v_\theta}{r_0}\right)_{r=r_0}. \tag{10}$$

Substituting the expressions for the stress tensors (9), (10), (4a) into (8), remembering that, for the small spheres considered in this problem, $p_0 = \text{const}$, and describing Ω_λ in Cartesian coordinates, in which $\Omega_y = \partial\chi/\partial z$, $\Omega_z = \partial\chi/\partial y$, we obtain

$$F = 2\pi r_0^2 \rho U \int_0^\pi \left\{\left[\frac{\partial\varphi}{\partial x}\cos\theta + \frac{\partial\varphi}{\partial y}\sin\lambda\sin\theta + \frac{\partial\varphi}{\partial z}\sin\lambda\sin\theta\right] + a_1 r_0^2 \sin^2\theta\cos\theta\right\}\sin\theta d\theta.$$

The integral of the second term is equal to zero, the first term is equal to $\partial\psi(r, \theta)/\partial r$. According to [5], quite independently of the number of terms considered in the expansion for φ, all the terms except the first will vanish on integration over the whole surface of the sphere; the first term will give

$$\iint_S \frac{\partial\varphi}{\partial r} dS = -\iint_S \frac{A_0}{r^2} dS = -4\pi A_0 \tag{11}$$

$$F = -4\pi\rho U A_0 \approx 6\pi\mu U r_0\left(1 + \frac{3}{8}\mathrm{Re}\right)\frac{2\mu + 3\mu_1}{3\mu + 3\mu_1} = \frac{\rho U^2}{2}\pi r_0^2 \frac{12}{\mathrm{Re}}\left(1 + \frac{3}{8}\mathrm{Re}\right)\frac{2\mu + 3\mu_1}{3\mu + 3\mu_1}.$$

Thus, as in the Stokes approximation, the resistance of the liquid sphere is determined by the product of the resistance coefficient of a solid sphere and a correcting factor depending on the viscosity of the medium inside and outside the sphere. It may be shown that the solution in the Ozeen approximation is suitable for Re < 1/2; for Re $\geqslant$ 1/2 comparison with experiment is required. For Re = 10 the approximate solution is 1.5 times as great as the experimental values of the resistance given in [7]. Figure 1 compares the resistance of a single bubble in an infinite liquid calculated by means of Eq. (11) ($C_D = F/(\rho U^2/2\ \pi r_0^2)$) with the generalized experimental data of [8]. For Re= 10 the calculated resistance is ~3 times greater than experimental. Under real conditions the resistance of the bubble to the motion may increase by up to another 1.5 times, owing to the presence of surface-active substances in the liquid [9]; a slight increase may also occur as a result of the presence of the wall close to the moving bubble [10]. However, for the boiling of an underheated liquid on a vertical wall the simultaneous motion of a large number of bubbles will lead to a fall in resistance. According to [11], the resistance of two solid spheres moving parallel to one another in a liquid, calculated on the Stokes approximation, falls by a factor of ~1.5 times as they are brought closer together. The presence of a spherical liquid drop causes less perturbation in the flow than a solid sphere of the same radius; in particular there is less vorticity of the flow, less resistance of the drop, the velocity of the liquid will be less behind the drop than behind a solid sphere, and the influence of other drops on the motion will be less than in the case of solid spheres. In our present case the intensity of the vortices will, to a first approximation, be determined by the following equation for r/r_0 > ~10.

$$\Omega = C_0 \frac{1+kr}{r^2} \sin\theta \exp\left[-kr(1-\cos\theta)\right], \tag{12}$$

i.e., at coincident points in space the vorticity will be one and a half times smaller for the motion of a gas bubble in a liquid than for the motion of a solid sphere. Thus in all cases the expressions for the resistance of a spherical bubble or drop, the velocities, and other perturbations of the surrounding liquid will constitute the product of several factors, each of which describe the influence of a particular phenomenon. This structure of the equations remains intact for large Re numbers as well; for example, according to [12] the resistance of an individual member of a large number of gas bubbles moving in a liquid will for 1 ≤ Re ≤ 300 be a factor of $(1-\varepsilon^{5/3})/(1-\varepsilon)^2$ times smaller (ε is the volumetric proportion of gas) than the resistance of a single bubble. Subsequently for calculating the lift height of the vapor bubble we shall use Eq. (11).

The rate of condensation of the vapor in the bubble for r_0 < 0.5 mm is determined by the rate of heat transfer from the surface of the bubble into the mass of liquid. The heat outflow was calculated using the interpolation equation

$$\mathrm{Nu} = 1 + \frac{1}{\sqrt{6\pi}} \mathrm{Pr}^{1/2}\mathrm{Re}^{1/2}.$$

The second term (according to [9]) describes the heat outflow from a small moving bubble for large Pr numbers, the first describes the heat outflow from a stationary bubble by way of molecular heat conduction. The distance which the vapor bubble travels before vanishing may be calculated from the equations

$$U\frac{dr_0}{dz} = \frac{\lambda(T_s - T_0)}{r_0\rho l}\left(1 + \sqrt{\frac{\mathrm{Pr}}{6\pi}}\sqrt{\mathrm{Re}}\right), \quad U = \frac{g}{3\nu} \cdot \frac{r_0^2}{1+\frac{3}{8}\mathrm{Re}}. \tag{13}$$

The results of the calculations are presented in Fig. 2. The calculations were carried out for thermophysical properties taken at 50°C: even for liquids differing very considerably from one another, both the lift velocity and the lift height remained almost exactly the same. The small relative velocities of the bubbles U ~ 1 cm/sec evidently explain the frequently mentioned fact that the process of surface boiling is independent of the velocity of the liquid [13-17].

Experiments were carried out in a tank 310 mm high and 300 mm in diameter. The walls were machined from AMG-6 aluminum alloy sheet 4.9 mm thick. The tank was filled to a height of ~270 mm with alcohol. The pressure in the "cushion" was 4.9 abs. atm, which corresponds to an alcohol saturation temperature of T_s = 125°C; pressurization was effected with argon. The heat flux to the side wall and the top of the tank was provided by an infrared radiator assembled from 193 KI-220-1000 lamps. The top of the tank was heated to T = 150°C before the experiment in order to avoid the condensation of alcohol vapor. The initial temperature of the alcohol was ~15°C. The heat flux was q = 500 kW/m^2 and 710 kW/m^2. During the experiments we measured the temperature of the outer walls of the tank with 10 thermocouples sited along two opposite generators at a distance of 45 mm from one another in the vertical direction, and the thermal flux with calorimeters; we also measured the temperature of the top (roof), the temperature at two points inside the tank at distances of 20 and 40 mm from the inner surface of the top, and the pressure inside the tank. We neglected the cross-leakage of the heat along the side walls from the top and bottom of the tank, since the effects of this were only perceptible at a distance of 10-12 mm. Under the experimental conditions (T_w–T_s) ~5° and the underheating was (T_s–T_0) ~ 110°. In order to study the life height of the vapor bubbles in the underheated liquid, we placed an annular screen or barrier at various heights in the tank (Fig. 3); this drew the bubbles away from the layer close to the wall and into the interior of the liquid. We see from Fig. 4 that for a distance from the annular barrier to the surface of the liquid H = 8-10 cm the rise in pressure in the "cushion" does not depend on this distance, i.e., the bubbles formed at a greater depth collapse in the liquid and never reach the surface of the alcohol. When the annular barrier was placed at a depth of ~1 cm the pressure increment in the "cushion" was two or three times smaller than when the barrier was situated at greater depths, i.e., a considerable proportion of the fairly large bubbles formed on the wall down to a depth of 5-8 cm reach the surface and contribute to the pressure rise; the lift height of the bubbles actually observed corresponds to the greatest initial radius of ~0.35-0.4 mm.

NOTATION

p, pressure; U, velocity of incident flow at infinity or lift velocity of the bubble; r, radius; v_r, v_θ, radial and azimuthal velocity components; ρ, density; Δ, Laplacian; g, gravitational acceleration; x, z, coordinate along the motion, displacement of the bubble; λ, thermal conductivity; T, T_s, T_w, T_0, temperature, saturation temperature, wall temperature, and temperature of the liquid a long way from the bubble or at the initial instant of time; l, specific heat of vaporization; c, specific heat; Re, Pr, Nu, Reynolds, Prandtl, and Nusselt numbers, respectively; H, lift height of the bubble.

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